



## Indeterminacy in the systems and networks design

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### Abstract

As on the stage systems and networks operation, part of the parameters which are taken into consideration in the design process change its meaning, the project formed earlier can not satisfy the requirements. It is necessary to respect a variability at the time of selected system or network characteristics in the design process. The authors propose a method introducing an indeterminacy to the design process: design criteria, input parameters and model itself. It allows to obtain not one but a set of design solutions which specify given requirements and it is possible to take a human factor into consideration in the solution search process.

### 1. Introduction

Systems and networks change incessantly their parameters in the evolution process. With reference to the systems these changes involve mainly increase of system capacity which is essential for solving of computational tasks with the higher and higher complexity. The changes observed in the networks are related to the increase of communication channel capacity. These changes allow to use the new services in the networks and those of multimedial transmission [1].

Until recently the basic method of capacity computer systems characteristics improvement was the increase of running rate elements base. With time it turned out that the results obtained in this way are insufficient. Therefore structural methods of capacity improvement based on using parallel processing found application. These methods, owing to combination of many computational elements with the aid of additional partition and task migration procedures allowed to increase computer system capacity. Along with the increasing frequency of process elements working in the computer systems, the new problem has appeared – communication problem between the processing elements. In the modern parallel systems, the information transmission times between the system components were surpassing delays resulting from the computation process [1].

In the case of computer networks the basic restrictions were a result of coarse-grained computational tasks granulation and thus of significant

communication operation cost. Moreover, multimedia services started to be used in the networks with significant demand for transmission band (transfer of sounds and mobile pictures).

The solution of transmission band deficit problem was the use of optical communication channels which guarantee information transfer at the significant speed at long distances. However, optical communication caused the appearance of new problems unknown until now. Optical channels have so huge capacity that it is impossible to use them by a small group of users. The solution of this problem is division of the physical channels into logical channels which are used by a small group. The usage of multi-channel allows to improve system security, simplify planning tasks and equalize loads etc [2,3].

But the usage of multi-channel requires modification of design procedure. If we use it there appear new tasks in the design project, for example assignment available communication channel to users. For this reason, appearance of new factors design process has yielded to complication. Moreover, a lot of parameters describing project undergo changes at time. Thus it is necessary to modify a design procedure so that it could guarantee creation of the system with given characteristics taking changes at operation time of selected solution parameters into consideration.

## 2. Indeterminacy in the design process

Let us consider, for example the computer network with a hierarchical structure. The basic levels of hierarchy are: network core level, access network level, user level. In traditional solutions the location and node assignment stage are realized at the time of design user network level. Thus, when designing network infrastructure we can encounter location problems [4-6]. Location problem is the problem of making the optimal choice in the spatial context. It means that problems involve tasks of placing a given number of devices in order to reach required property such as minimization fixed costs or distance function between devices. The classic algorithms of interconnection architectures design are multistage and based on composition of interconnection network multilevel. The traditional model describing the above problem is as follows:

$$\min \sum_{i \in N} \sum_{j \in M} C_{ij} x_{ij} + \sum_{j \in M} F_j y_j$$

subject to

$$\begin{aligned} \sum_{j \in M} x_{ij} &= 1 \quad \forall i \in N, \\ x_{ij} &\leq y_j \quad \forall i \in N, j \in M, \\ x_{ij} &\in \{0,1\} \quad \forall i \in N, j \in M, \\ y_j &\in \{0,1\} \quad \forall j \in M, \end{aligned}$$

where  $C_{ij}$  denotes the cost of assignment terminal  $I$  to the concentrator  $j$ ,  $F_j$  denotes the cost of installing a concentrator at the the location  $j$ ,  $y_j$  equals 1 if a concentrator is installed at location  $j$  or 0 otherwise, for all  $j \in M$ . Similarly, parameter  $x_{ij}$  equals 1 if the terminal  $i$  is assigned to the concentrator at the location  $j$  or 0 otherwise, for all  $i \in N$  i  $j \in M$  [4].

The presented model is very simplified and does not express problem complexity. It does not include many parameters describing real conditions. Therefore, it is necessary to take into consideration indeterminacy of parameters in order to optimize solution of location problem. These parameters can be linked, for example to traffic flow in the network, changes of flowability, quantity of users, network elements failure, etc.

### 3. Mathematical object model in the indeterminacy conditions

Let us consider the design process multi-channel optical interconnection network. Let us assume that the design process consists of two stages. In the first stage there will be denoted relative values of parameters we are interested in, which are selected so that they could describe a designed object. The values of selected parameters are compared for each topological organization. In the second stage on the basis of below survey there is denoted a scaling factor which allows converting relative values into absolute. Application of the above methodology permits denoting an essential set of model parameter values. It is possible also to get concrete values of essential parameters with changeable knowledge level of designing project. If there appears additional information in the research process describing requirements for the object, which can denote more exactly model parameters in particular, they can be taken into consideration in model as additional estimates at the described time of relative parameter values.

In the above method the particular part is the way which allows to combine the whole set of varied estimates with each other and to present them as a uniform numerical parameter denoting relative value of parameter. This method is one of the most important elements of the proposed design procedure. With this end in view we can modify the methods presented in section [7]. The studied method is designed for solving a wide class of solution search tasks in the indeterminacy conditions, when from the set  $Y$  of the task solution variants there is selected the best one. In the above method the basic type of indeterminacy is criterion indeterminacy. It involves that efficiency of design problem solution variant can not be described by one numerical value and requires a whole set of numbers, each set denotes one of the concrete efficiency aspects.

In this case only connection of all estimates in the universal way evaluates characteristics of specific object version. In this way there is used the vector of optimality criterion instead of single optimality criterion  $f(y)$ :

$$f(y) = (f^1(y), f^2(y), \dots, f^N(y)). \quad (1)$$

The next type of indeterminacy is the source data indeterminacy, when in spite of chosen variant  $y$  and completely known source data also source data whose accurate value is unknown at the time of denoting variant have influence on the value of criterion vector. For these data it can be only shown a value range that is a set  $X$  and the vector  $x$  of real data also belong to it. This set will be called as the indeterminacy set.

Given the criterion and source data indeterminacy efficiency of variant  $y$  can be determined by the criterion vector:

$$f(x, y) = (f^1(x, y), f^2(x, y), \dots, f^N(x, y)) \quad x \in X. \quad (2)$$

Note that from a mathematical point of view any indeterminacy presented above can be shown by means of another one.

The third type of indeterminacy is model indeterminacy which assumes that the principle of counting optimality criterion values is inaccurate. Using the correction factors of values that are in the range of model accuracy it can be converted to the source data indeterminacy.

In this way it is necessary to extend a classic concept of solutions optimization by introducing the set of indeterminacy  $X$  beside the set of acceptable solution variants  $Y$ , which reflects criterion, source data and model indeterminacy. Then the optimality criterion  $f(x, y)$  will be described on the product of set  $XY$ , it does not allow to choose the best unique solution by its optimization. It results from the fact that each value from the set  $X$  does not correspond to its variant of the best solution obtained by optimization  $f(x, y)$  on the set  $Y$  for a given  $x$ .

There is difference between the tasks of acceptance solution and optimization tasks in the presented formal unclosed. The unclosed of task requires a direct share of man or another decision factor in the describing solution. The decision factor (a man) can introduce additional information that allows to eliminate indeterminacy or chooses a method of determining solution in the indeterminacy conditions.

In most methods which are used in making decisions change from local to global consideration of indeterminacy influences on efficiency of varied solution variants. This principle will be called the way of respecting indeterminacy. From the mathematical point of view this way is that for fixed  $y$  from the set  $Y$ . It contrasts the number  $F(X, y)$  with the function  $f(x, y)$  described on the set  $X$ .

If the  $f(x, y)$  criterion is called generalization loss, then  $F(X, y)$  should be determined as  $H$ -generalization losses, i.e. generalization losses described taking indeterminacy into consideration.  $H$ -generalization losses can be described as

the average value of generalization losses on the set  $X$  or the largest value of losses on this set. There are other acceptable ways of respecting indeterminacy.

If the way of respecting indeterminacy was determined the task of acceptation decision is over. Then  $H$ -generalization losses are described on the set of acceptable solutions  $F(X, y)$ ,  $y \in Y$ , i.e. the best solution can be found by means of classic optimization  $f(X, y)$  on the set  $Y$ .

Note that in the stage of forming decision task, the way of respecting indeterminacy is unknown, otherwise a solution would be the classic optimization task. You can assume that there exists a certain set  $S$  of acceptable ways of respecting indeterminacy and the decision factor derived from it should choose a way adequate to the task solved. In principal, it is not essential in which way the given set is described. To this end you can use: calculation of the ways of respecting indeterminacy, a list of properties, an intuitive imagination of factor making a decision. However, in each case the solution choice involves searching the way of respecting indeterminacy by the decision factor.

There exist lots of making decision methods that do not take a notion of the way of respecting indeterminacy into consideration but it is not advisable to use them in this case. The way of respecting indeterminacy allows to introduce a human factor to the process of making decision thus the basic task is run by computer. Moreover, there is kept a stable numerical basis in all the stages of choosing solution [8].

In this way, the presented model of making decision consists of three sets: acceptable solution variants  $Y$ , indeterminacy  $X$ , acceptable ways of respecting indeterminacy  $S$  and function of generalization losses  $f(x, y)$ , which is the local generalization optimality criterion. Without constraints on generality this function can be considered as a normalized one:

$$0 \leq f(x, y) \leq 1. \quad (3)$$

In these contributions the choice of the best solution becomes the decision factor preferential right and amounts to:

1. minimization indeterminacy taking subjective information of decision factor if the latter considers that as purposeful,
2. choice one of the ways of respecting indeterminacy in order to make a final decision in the indeterminacy conditions,
3. application of the selected way of respecting indeterminacy and estimation of the obtained result.

An universal respecting indeterminacy requires:

1. description of the whole set acceptable ways of respecting indeterminacy,
2. determination of the subset the best ways of respecting indeterminacy from it, that will be used for choosing a solution.

Now the proposed method will be described in detail.

#### 4. Optimization model in the indeterminacy conditions

There was introduced the outer set  $X$  (the set of elementary tasks  $x \in R_n$ ) and the set of strategies  $Y \in R_p$  in this model. The connection of  $m$  optimized objects (elements)  $y_i \in Y$  ( $i = 1, 2, \dots, m$ ) is called the strategy  $A$

$$A = \{y_i\} \subset Y \quad i = 1, 2, \dots, m. \quad (4)$$

An unique integer function of distribution  $E(x)$  is determined on the set  $X$  and it takes values of  $1, 2, \dots, m$ . In this way on the set  $X$  you can determine its specialization range  $D_i$  for each element  $y_i \in A$  ( $i = 1, 2, \dots, m$ ), when the function of distribution takes the value equal  $i$ :

$$D_i = \{x \in X / E(x) = i\} \quad i = 1, 2, \dots, m. \quad (5)$$

It is obvious that specialization ranges do not overlap and their connection denotes the outer set:

$$D_i \cap D_k = \emptyset \quad \forall i, k = 1, \dots, m, \quad \bigcup_{i=1}^m D_i = X. \quad (6)$$

The notion  $(X, A, E(x))$  is called a system with many features. Additionally, there were described the vector functions:

$$f = f(x, y, M(D_{E(x)})), \quad f = (f^1, f^2, \dots, f^s), \quad (7)$$

where: the components  $f^1, f^2, \dots, f^s$  were called the local efficiency indicators of doing tasks  $x \in X$  by the object  $y$  of strategy  $A$ , and they are denoted by the values  $x \in X$ ,  $y \in Y$  and the measure  $M(D_{E(x)})$  of specialization range (it is assumed that the set  $Y$  is a measurable set) to which the element  $x \in X$  belongs.

Functions (7) are determined for any  $y \in Y$ ,  $M(D_{E(x)}) \geq 0$  but not for any  $x \in X$ . The range of their determination on the set  $X$  will be denoted as  $d(y)$  and called the attainable tasks range. Then it is necessary to satisfy the requirements:

$$D_i \subset d(y_i) \quad \forall i = 1, \dots, m, \quad y_i \in Y. \quad (8)$$

In the real applications there are the three special cases described above:

$$d(y) = X \quad \forall y \in Y, \quad (9)$$

i.e. functions (7) were determined on the whole set  $X$ :

$$f = \rho(x, y), \quad (10)$$

i.e. function (7) transforms into the function form:

$$f = \rho(y_i, \mu_i), \quad (11)$$

where:  $\mu_i = M(D_i)$ .

For the fixed strategy  $A \in Y$  and the distribution function  $E(x)$  we determined a notion of system efficiency vector with many features:

$$F = F(X, A, E(x)), \quad F = (F^1, F^2, \dots, F^s) \quad (12)$$

and the rule  $\phi$  of determination of its components by the values of local efficiency indicators for a pessimistic estimation:

$$\phi_1(y) \equiv F(X, y) = \max_{x \in X} f(x, y); \quad (13)$$

averaging the (integral) estimation:

$$\phi_2(y) \equiv F(X, y) = \frac{1}{n} \sum f(x, y) \quad \text{for } X = \{x_i\}, \quad i = 1, 2, \dots, n. \quad (14)$$

The optimization task MC consists in choosing the characteristics  $E(x)$  and  $A = \{y_i\} \subset Y, \quad i = 1, \dots, m$  optimally from the minimization of the whole MC efficiency vector point of view (according to [7], the notion of optimization requires a constraint). There are considered the following cases:

1. the outer set  $X$  and the strategy  $A$  were determined, it is necessary to find optimum distribution function  $E(x)$  – the task of optimum distribution:

$$F(X, A, E(x)) = \min_{E(x)} F(X, A, E(x)); \quad (15)$$

2. the outer set  $X$  and the number of elements  $m$  strategies were determined, it is necessary to find the optimum strategy  $A$  and the optimum distribution function – the task of strategy optimization:

$$F(X, A, E(x)) = \min_{\substack{E(x), \\ A \subset Y, \\ m = \text{const}}} F(X, A, E(x)); \quad (16)$$

3. the outer set  $X$  was determined, it is necessary to find the optimum strategies  $A$  and the optimum distribution function  $E(x)$  – the task of generalized optimization:

$$F(X, A, E(x)) = \min_{\substack{E(x), \\ A \subset Y}} F(X, A, E(x)). \quad (17)$$

Let us consider a system model with many features. It is obvious that the outer set  $X$  that is in it reflects a variety of tasks and system purposes. Next, the existence of efficiency indicators vector  $F = (F^1, F^2, \dots, F^s)$  points at a variety of purposes.

Strategies  $A$  consist of  $m$  elements –  $y_i \quad (i = 1, \dots, m)$  whose co-operation consists in division of tasks which form an outer set between them. This distribution is determined by the function  $E(x)$  in this way that the specialization range  $D_i$  defines an effect zone for each element of strategy  $A$ . Thus, each strategy  $A$  describes a special variant of system with many features.

The technical object  $y$  can not realize any tasks  $x \in X$  (for example, access node is not allowed to accept larger streams than its capacity). This property is defined in the model by the task ranges  $d(y)$ . Satisfying constraint (8) guarantees that strategy  $A$  assures completion of all tasks  $x \in X$ .

Functions (7) efficiency of task  $x \in X$  realization by the element  $y_i \in A$  depend on the measure  $\mu_i$  of proper specialization ranges. It allows to extend the range of model applications. For example, economical efficiency of connection system is defined not only by its characteristics and parameters, but also by versatility of its application. There are obvious modifications of the described mathematical model, oriented at simplification of initial problem definition. In this way, the idea of proposed method of optimal system design with many features leads to defining the outer constraint set and optimizing multi-element design solution in the optimum set.

With reference to the design object as the element of some system, the set of outer constraints describes a performed operation set, the system describes a solution of multi-elementariness and a set of solutions – an acceptable set of system parameters. An optimization consists in dividing the set of outer constraints into ranges of the most effective application of independent objects, and also – in the choice of optimum parameters.

With reference to the design process the set of outer constraints describes indefinite factors which can be divided into removable and unremovable in the design process, while solving multi-elementariness – competitive variants of design object. The first type of indeterminacy consists of those that can not be removed partly or completely until receiving a solution of problem and also in the operating process, for example, uncertainty about information on real construction and system operation constraints. Unremovable indeterminacies are described by introducing an outer set and solving one of the optimization tasks defined by expressions (15)-(17).

The second type is the connection of indeterminacies which can be removed before forming a project solution by physical modeling, testing and experiences. They are connected by operators inaccuracy  $\varphi(x, y)$  and  $F(X, y)$  describing the object and its efficiency.

While describing the design process an acceptable set of project solutions and solution class changing in time is considered. In this case with the appearance of each new solution, the domination ranges are converted by one of the optimum distribution methods. Consequently, a propriety of the new solution with varied deformations of acceptable project solutions set is made.

In the proposed approach many rules of solutions estimation in the indeterminacy contributions of source data, models, tasks and criteria were introduced. Some properties of principle set are described by some axiom-based and they allow to calculate a solution efficiency indicator taking an

indeterminacy set into account. The interpretation of criteria connection in the form of methods indeterminacy (inaccuracy) of efficiency estimation allows to reduce the problem of choosing a solution to optimization of multi-element solution optimum covering method-based.

In order to realize the proposed method the optimization methods and algorithms were worked out. They are based on the idea of dividing the outer set into Dirichlet's ranges and taking a system efficiency indicator into account.

Formalized creation of the mathematical models of forms of technical compound objects includes a dialog procedure of denoting the sufficient and non-inverted set of models on the basis of analysis of informative graph and formulation of the optimum model algorithm in the sense of computation.

In the first stage there is formed an informative model graph which is the unclosed system of nonlinear equations and constraints in its basis.

In order to create the graph, there is analyzed a physical sense of the model formulas bank in the dialog mode and its changes are divided into dependent and independent ones. The basic properties of objects are checked in the model content:

1. requirements of making determined task set,
2. the principle of object indivisibility,
3. physical principles of directed object functional,
4. expected technical level.

Next on the basis of analysis of formal graph structure, the closed equations and constraints system is separated and there is created algorithm of its solution with the constraint of minimization calculation time, which synthesizes acceptable solution variants.

## 5. Conclusions

To sum up, the following conclusions can be drawn.

1. We have started work on this subject because of problems with effective meeting users' needs in the stage of wide area computer networks designing process. The analyses showed that this problem can not be solved using the existing methods and resources.
2. Considering indeterminacy in a few different ways in the decision process we have to take the decision factor into account. We have to make some formal method that allows to make decision including a variety of indeterminacy methods satisfying the minimal use of the decision factor. This method is presented in this paper.
3. The decision problem including indeterminacy conditions can be described with the three sets: possible solution, indeterminacy and possible methods of indeterminacy application. We can accept the way of indeterminacy satisfying the conditions: monotonic, stability, versatility and smoothness.

4. The set of accepted ways of satisfying indeterminacy is isomorphic to generating function sets – continuously, monotonically increasing functions with one variable  $G(t)$ , fulfil the conditions:  $G(0)=0$ ,  $G(1)=1$ .
5. The relationship given in this paper allows on the basis of general equations to define H – generalization losses for any creating function, i.e. for any used way to satisfy indeterminacy. For multicriterial tasks we present the relationship that allows to avoid the use of multidimensional integral in the design process.
6. For the multicriterial design tasks we can get complete dependence that allows to determine H-generalization losses for a typical set of ways satisfying indeterminacy.
7. The use of this method in designing the network process and in the resources location process in wide area computer networks is shown in papers [9-11].

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