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# Nonlinear weighted median filters in dyadic decomposition of images

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**Abstract**

There are many techniques of loss image compression based on dyadic decomposition of images. In these decompositions always linear filters are used. In this paper it is proposed to use nonlinear weighted median filters in the place of linear ones and it is shown how median filters may be used in dyadic decomposition. This technique has been compared with the non-standard wavelet decomposition, in particular with the Haar wavelet filtering. The experiments made show that in the loss image compression case using median filters gave better visual quality of the reconstructed images.

## 1. Introduction

Dyadic image decomposition is frequently used in the image analysis, especially in loss and lossless image compression [1-4]. For example, the most promising standard of image compression – JPEG2000 [2] uses the wavelet decomposition as a main power. The dyadic wavelet decomposition is frequently used in image processing because of its speed, simplicity and good properties (catching changes of signal in both frequency and time) in still image compression. However, using linear filtering in image processing, although it is simple in computer implementation, does not often give satisfactory quality of images. It turns out that, from the image compression point of view, substituting the linear filters, as wavelet filters are, with nonlinear ones one can prove the quality of reconstructed images, preserving the same compression ratio.

To get the possibility of properly using filters in still image compression these filters must fulfil, the so-called, perfect image reconstruction condition [4]. It means, in short, that these filters must be reversible. That is after forward filtering we get the spectrum image and from it we should have the possibility of getting back the original image without any distortions, applying for example the

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inverse filters. It is not so difficult to get such linear filters (based for example on Haar, Daubechies and other wavelets [4]). But in the nonlinear case such process is not so simple. After forward non-linear filtering often we cannot get the original image back. Though, as will be shown in this paper, applying some improvements to the nonlinear filters – weighted median in this case, we can get the perfect image reconstruction condition, retaining the non-linearity of these filters. It will be also shown how such improved filters may be used in dyadic image decomposition in a similar way as the Haar wavelet filters.

## 2. Non-standard dyadic wavelet decomposition

The dyadic wavelet decomposition, especially in the Haar wavelet case, is the well-known theory [3, 4] and will not be presented here in detail. Only the main ideas will be presented for better understanding of further considerations and showing the similarities and analogies between this technique and the new-presented one.

Let us denote **F** as an image in the square matrix (of degree $N=2^n$) form and define the filter operators used in dyadic decomposition of **F**. In the Haar wavelet case the operators are defined as follows

$$\begin{aligned} \mathbf{L}(i) &= \frac{1}{\sqrt{2}}\mathbf{F}(2i) + \frac{1}{\sqrt{2}}\mathbf{F}(2i+1), \\ \mathbf{H}(i) &= \frac{1}{\sqrt{2}}\mathbf{F}(2i) - \frac{1}{\sqrt{2}}\mathbf{F}(2i+1), \end{aligned} \tag{1}$$

where:

$\mathbf{F}(i)$ - vector of size *N*, containing row or column of matrix **F**, $i \in \{0,1,\ldots,\frac{N}{2}-1\}$,

$\mathbf{L}(i)$ - vector of size *N/2*, containing approximation coefficients,

$\mathbf{H}(i)$ - vector of size *N/2*, containing detail coefficients.

The **L** and **H** are the so-called low pass and high pass filters, respectively.

To get non-standard wavelet dyadic decomposition on the first level of an image **F** – the spectrum matrix called $\mathbf{S}_1$ – we first apply the operators (1) to all columns of the matrix and then to all rows [5]. Then, to get the second level of non-standard decomposition – matrix $\mathbf{S}_2$ – one can apply similar analysis to the upper left sub-matrix of size $\frac{N}{2} \times \frac{N}{2}$ of matrix $\mathbf{S}_1$, and so on, till *n*-th ($n = \log_2 N$) level of decomposition.

Because filters (1) fulfil perfect image reconstruction condition [4], we can also define similarly the inverse operators

$$\begin{aligned} \mathbf{F}(2i) &= \frac{1}{\sqrt{2}}\mathbf{L}(i) + \frac{1}{\sqrt{2}}\mathbf{H}(i), \\ \mathbf{F}(2i+1) &= \frac{1}{\sqrt{2}}\mathbf{L}(i) - \frac{1}{\sqrt{2}}\mathbf{H}(i), \end{aligned} \tag{2}$$

where $\mathbf{F}(i), \mathbf{L}(i), \mathbf{H}(i)$ denote vectors as above in (1) and $i \in \{0,1,\ldots,\frac{N}{2}-1\}$.

Applying these filters to the spectrum image allows us to get the original image back without any distortions.

## 3. Weighted median dyadic decomposition

Let us denote $Median(x_1, \ldots, x_n)$ as the median of integer elements $x_1, \ldots, x_n$, $n \in ¥$. Furthermore, the notation $k\Diamond x$ denotes that element $x$ occurs $k$ times – $k$ is the so-called weight, that is for example we can define the weighted median of four elements as

$$Median(2\Diamond x_1, 2\Diamond x_2, x_3, x_4) = Median(x_1, x_1, x_2, x_2, x_3, x_4).$$

Similar to the wavelet decomposition case, we can define nonlinear – weighted median filters, which may be used in dyadic decomposition of an image. The filter formulas are as follows

$$\begin{aligned}
&\mathbf{A}(i,j) = \mathbf{F}(2i,2j),\\
&\mathbf{D}(i,j) = \mathbf{F}(2i+1,2j+1) - Median(\mathbf{A}(i,j),\mathbf{A}(i,j+1),\mathbf{A}(i+1,j),\mathbf{A}(i+1,j+1)),\\
&\mathbf{H}(i,j) = \mathbf{F}(2i,2j+1) - Median(2\Diamond\mathbf{A}(i,j),2\Diamond\mathbf{A}(i,j+1),\mathbf{D}(i,j),\mathbf{D}(i-1,j)),\\
&\mathbf{V}(i,j) = \mathbf{F}(2i+1,2j) - Median(2\Diamond\mathbf{A}(i,j),2\Diamond\mathbf{A}(i+1,j),\mathbf{D}(i,j),\mathbf{D}(i,j-1)),
\end{aligned} \tag{3}$$

where:

$\mathbf{F}$ – image in the matrix form of the size $N \times N$, $i, j \in \{0,1,\ldots,\frac{N}{2}-1\}$,

$\mathbf{A}$, $\mathbf{V}$, $\mathbf{H}$, $\mathbf{D}$ – matrices of size $\frac{N}{2} \times \frac{N}{2}$, containing the median coefficients.

Note, that of all four filters only the **A** filter is linear (but note, that this filter applied alone is not reversible). As a result of its filtering we get the decimated copy of the original image. The filters **V**, **H**, **D** are non-linear and as a result of their filtering we get differences between the appropriate original pixels and their median approximations. Note, that the median approximations in this case mean zooming 200% of the original image decimated earlier.

Similar to the Haar wavelet decomposition case, presented in the previous section, we can apply filters (3) to an image to get the first level of dyadic decomposition. More precisely this process looks as follows. Consider the sample fragment of the original image depicted in the matrix form as shown in Fig. 1A. Let us denote, for simplicity, the original image pixels as ***a*** (they form only the mask, not the real values of the pixels). First we remove every other pixel in horizontal and vertical directions – see Fig. 1B. Next, using the median filter, we approximate the pixels denoted as ***d*** in the figure (step 2) using four corner pixels ***a***. Because all four corner pixels ***a*** are original – they are equally trustworthy, so all have weights equal to 1 in the median definition (3). Next, having all pixels ***a*** and ***d***, we approximate the pixels ***h*** and ***v*** (step 3). But, this time we approximate them using also pixels ***d***, which are not original image pixels. So the pixels *a*, as more trustworthy, have larger weights in the median definition – equal to 2. Next, having approximated all image pixels, we compute the difference between the original image and the approximated one, but only on *d*, *h* and *v* pixels, remaining pixels *a* without change (see Fig. 1E). And finally, we get the spectrum image by setting special order of the pixels as shown in Fig. 1F. Thereby we get the image containing four sub-images **A**, **D**, **H** and **V** containing respectively: original image pixels (but decimated in horizontal and vertical directions), diagonal approximations, horizontal and vertical weighted approximations. Thus we get the image on the first level of decomposition, which has similar properties to the Haar wavelet one. To get the decomposition on the next levels, we proceed similarly to the wavelet case – apply this analysis successively to the upper left corner of the matrix, till *n*-th ($n = \log_2 N$) level of decomposition.

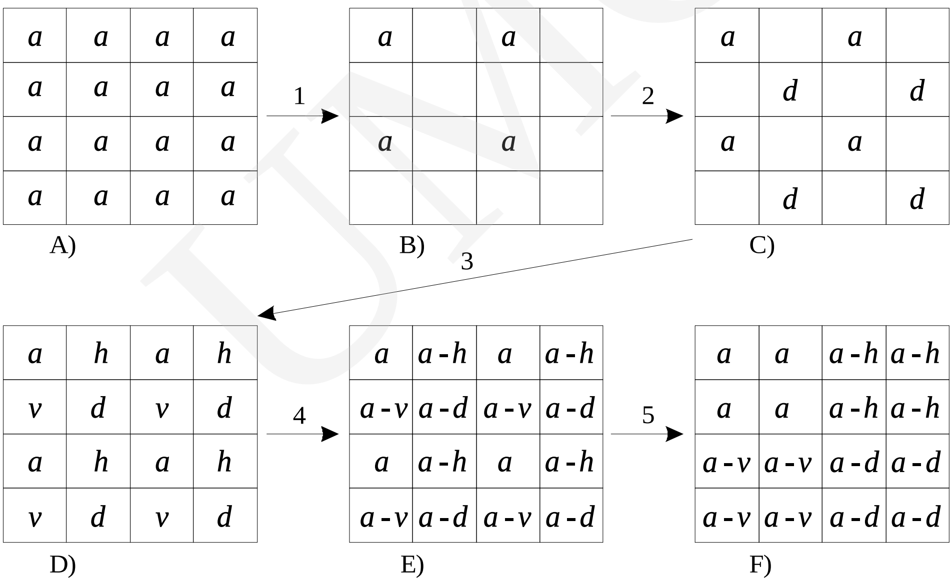



Fig. 1. The scheme of dyadic median decomposition

Note, that such defined filters, even though they are non-linear, fulfil the perfect image reconstruction condition. Indeed, applying inverse filters (4) to the spectrum allows us to get back the original image. The inverse filters are defined as follows

$$\begin{aligned}
&\mathbf{F}(2i,2j) = \mathbf{A}(i,j),\\
&\mathbf{F}(2i+1,2j+1) = \mathbf{D}(i,j) + Median(\mathbf{A}(i,j),\mathbf{A}(i,j+1),\mathbf{A}(i+1,j),\mathbf{A}(i+1,j+1)),\\
&\mathbf{F}(2i,2j+1) = \mathbf{H}(i,j) + Median(2\Diamond\mathbf{A}(i,j),2\Diamond\mathbf{A}(i,j+1),\mathbf{D}(i,j),\mathbf{D}(i-1,j)),\\
&\mathbf{F}(2i+1,2j) = \mathbf{V}(i,j) + Median(2\Diamond\mathbf{A}(i,j),2\Diamond\mathbf{A}(i+1,j),\mathbf{D}(i,j),\mathbf{D}(i,j-1)),
\end{aligned} \tag{4}$$

where **F**, **A**, **D**, **H** denote matrices as above in (3) and $i, j \in \{0, 1, \ldots, \frac{N}{2} - 1\}$.

These filters work similarly to the forward (3) ones. First, from the upper left sub-image of the spectrum (Fig. 1F) we construct – expand image as shown in Fig. 1B. Next we repeat steps in turn 2, 3 and 4 but substituting subtraction of matrices 1A and 1D from step 4 by the addition of matrices 1D and 1E. As a result of these operations we get the original image (Fig. 1A) without any distortions.

## 4. Experimental results

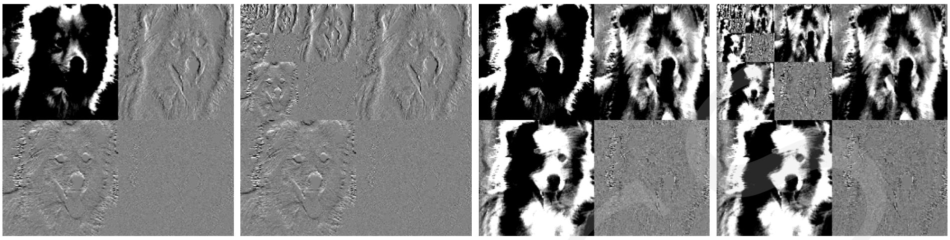
In short, to loss compress an image using dyadic decomposition we must first apply some filters to the original image to get the spectrum. Next we set some elements (very often those low in magnitude) of the spectrum to zero. There are many techniques to set these elements to zero [6]. Finally, we compress such spectrum image lossless using some coding (for example Huffman coding [4]). To decompress the image we first decode the data to get the spectrum with zero entries and apply the inverse filters to it to reconstruct the image. The reconstructed image has some distortions because of setting some elements of the spectrum to zero. Thus, which and how many elements we set to zero is of enormous significance.

Experiments presented in this paper were concentrated only on applying filters (forward and inverse) to an image or spectrum and setting some elements of spectrum to zero to worsen the quality of further reconstructed images. The problem of lossless spectrum image coding is well known [1, 4] so it will be skipped here.

The experiments were performed on a number of well known images. In this paper only simple examples will be presented. In Fig. 2 there is presented "collie" – one of the tested images.

Fig. 2. One of the tested images – "collie"

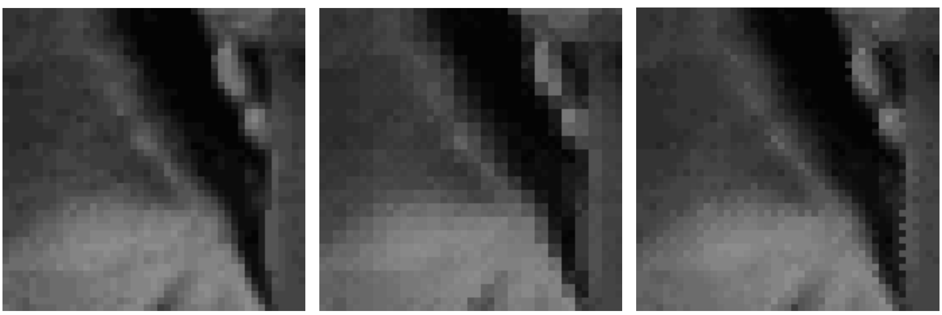
In Fig. 3 there are presented the well-known wavelet and the new – median decomposed images respectively on the first and last levels of decomposition.



a) b) c) d)

Fig. 3. Decomposed image: a) wavelet – first level; b) wavelet – last (8-th) level; c) median – first level; d) median – last (8-th) level

Note, that horizontal and vertical coefficients in the median decomposed image have larger magnitude than in the wavelet one. It comes from the fact, that these coefficients were approximated using not only original but also approximated earlier diagonal coefficients. However, removing them from the spectrum (that is setting them to zero) does not distort the reconstructed image more than in the wavelet case. Indeed, it is the well-known fact [7], that non-linear filtering gives better visual results than the linear one. Look for example at Fig. 4. In that figure there are fragments of the original (fragment of the dog's jaw) and reconstructed sample images all zoomed 500% to see some differences better. In both spectra of this original image the same amount of coefficients (exactly the half – all horizontal and vertical coefficients) has been removed. As seen in the pictures, the median-reconstructed image looks slightly better – the wavelet-reconstructed image loses some details, moreover one can notice clear decrease of its resolution.



a) b) c)

Fig. 4. The fragments of images: a) original; b) wavelet-reconstructed; c) median-reconstructed

Table 1. Quality of reconstructed images

| Image \ Quality | PSNR | | MSE | |
|---|---|---|---|---|
| | Wavelet | Median | Wavelet | Median |
| Collie (50%) | 31.827 | 33.037 | 42.696 | 32.311 |
| Lena (25%) | 29.961 | 30.559 | 65.617 | 57.167 |
| Cat (75%) | 31.567 | 31.935 | 45.332 | 41.643 |

In Tab. 1 there are a few sample results concerning the experiments. Beside the names of images the percent values denote the amount of removed spectral coefficients. From the spectra – wavelet and median the spectral coefficients situated in appropriate sites have been removed. Moreover, note that because we removed the same amount of coefficients we can get a similar compression ratio. As follows from that table the median-reconstructed images have better quality than the wavelet-reconstructed ones. That is, they have larger PSNR and smaller Mean Square Error.

## 5. Conclusions

The experiments showed that replacing linear filters (wavelet in this case) with nonlinear ones (median), allows to get better visual quality of reconstructed images in the loss compression case. Moreover, note that median filtering has some advantages. For example, in the median decomposed image on the first level of decomposition, there are two similar images, similar to the case of wavelet one. This similarity may be used to optimize the process of loss or even lossless image compression. Furthermore, significant similarity, different in scale, between proper images on different levels of decomposition also may be used to optimize the process – for example we can try to use loss fractal compression techniques [2, 8].

The Haar wavelet filters described in this paper have been chosen in our analysis because nowadays they are the most common ones of all linear filters due to their effectiveness. However the nonlinear weighted median filters used in the paper were selected since they are very promising techniques of image processing. Currently they are considered to give very good results in image zooming [7] (which has been exactly used in the paper in dyadic decomposition). Such combination of dyadic decomposition of images with median filtering seems to give best results if we want to join the best image quality with the large compression ratio.

## References

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