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Two hierarchies of **R**-recursive functions

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Abstract

In the paper some aspects of complexity of \mathbf{R} -recursive functions are considered. The limit hierarchy of \mathbf{R} -recursive functions is introduced by the analogy to the μ -hierarchy. Then its properties and relations to the μ -hierarchy are analysed.

1. Introduction

The classical theory of computation deals with the functions on enumerable (especially natural) domains. The fundamental notion in this field is the notion of a (partial) recursive function. The problem of hierarchies for these functions is also in the interest of mathematicians (for elementary, primitive recursive function, Grzegorczyk hierarchy, compare [1].

During past years many mathematicians have been interested in creating analogous models of computation on real numbers (see for example Grzegorczyk [2], Blum, Shub, Smale [3]). An interesting approach was given by Moore. In the work [4] he defined a set of functions on the reals \boldsymbol{R} (called \boldsymbol{R} -recursive functions) in the analogous way to the classical recursive functions on the natural numbers \boldsymbol{N} . His model has a continuous time of computation (a continuous integration instead of a discrete recursion). The great importance in Moore's model has the zero-finding operation μ , which is used to construct μ -hierarchy of \boldsymbol{R} -recursive functions.

It was shown [5] that the zero-finding operator μ can be replaced by the operation of infinite limits. This allows us to define a limit hierarchy and relate it to μ -hierarchy.

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2. Preliminaries

We start with a fundamental definition of a class of real functions called \mathbf{R} recursive functions [4].

Definition 2.1 The set of \mathbf{R} -recursive functions is generated from the constants 0,1 by the operations:

- 1) composition: $h(\overline{x}) = f(g(\overline{x}))$;
- 2) differential recursion: $h(\overline{x},0) = f(\overline{x}), \partial_y h(\overline{x},y) = g(\overline{x},y,h(\overline{x},y))$ (the equivalent formulation can be given by integrals: $h(\overline{x},y) = f(\overline{x}) + \int_0^y g(\overline{x},y',h(\overline{x},y'))dy'$);
- 3) **m**-recursion $h(\overline{x}) = \mathbf{m}_y f(\overline{x}, y) = \inf\{y : f(\overline{x}, y) = 0\}$, where infimum chooses the number \mathbf{y} with the smallest absolute value and for two \mathbf{y} with the same absolute value the negative one;
- 4) vector-valued functions can be defined by defining their components.

Several comments are needed to the above definition. A solution of a differential equation need not be unique or can diverge. Hence, we assume that if **h** is defined by a differential recursion then **h** is defined only where a finite and unique solution exists. This is why the set of **R**-recursive functions includes also partial functions. We use (after [4]) the name of **R**-recursive functions in the article, however we should remember that in reality we have partiality here (partial **R**-recursive functions).

The second problem arises with the operation of infimum. Let us observe that if an infinite number of zeros accumulates just above some positive *y* or just below some negative *y* then the infimum operation returns that *y* even if it itself is not a zero.

In the papers [5, 6] it was shown that if in the Moore's definition [4] m operation is replaced by infinite limits: $h(\overline{x}) = \liminf_{y \to \infty} g(\overline{x}, y)$, $h(\overline{x}) = \limsup_{y \to \infty} g(\overline{x}, y)$ then the resulting class of functions remains the same.

This gives us also the following result (including the limit operation in the form $h(\overline{x}) = \lim_{y\to\infty} g(\overline{x}, y)$, which can be in the obvious way obtained from limsup, liminf:

Corollary 2.2 The class of **R**-recursive functions is closed under the operations of infinite limits: $h(\overline{x}) = \liminf_{y \to \infty} g(\overline{x}, y)$, $h(\overline{x}) = \limsup_{y \to \infty} g(\overline{x}, y)$.

3. Hierarchies

The operator m is a key operator in generating the R-recursive functions. In a physical sense it has a property of being strongly uncomputable. This fact suggests creating a hierarchy, which is built with respect to the number of uses of m in the definition of a given f.

Definition 3.1 ([4]) For a given **R**-recursive expression $s(\overline{x})$, let $M_{x_i}(s)$ (the **m**-number with respect to x_i) be defined as follows:

$$M_x(0) = M_x(1) = M_x(-1) = 0,$$
 (1)

$$M_{x}\left(f\left(g_{1},g_{2},...\right)\right) = \max_{i}\left(M_{x_{j}}\left(f\right) + M_{x}\left(g_{j}\right)\right),\tag{2}$$

$$M_{x}\left(h=f+\int_{0}^{y}g(\overline{x},y',h)dy'\right)=\max\left(M_{x}(f),M_{x}(g),M_{h}(g)\right),\tag{3}$$

$$M_{y}\left(h=f+\int_{0}^{y}g\left(\overline{x},y',h\right)dy'\right)=\max\left(M_{y'}\left(g\right),M_{h}\left(g\right)\right),\tag{4}$$

$$M_{x}\left(\boldsymbol{m}_{y}f\left(\overline{x},y\right)\right) = \max\left(M_{x}\left(f\right),M_{y}\left(f\right)\right) + 1,\tag{5}$$

where x can be any $x_1,...,x_n$ for $\overline{x} = (x_1,...,x_n)$

For an **R**-recursive function f, let $M(f) = \max_{x_i} (s)$ minimized over all expressions s that define f. Now we are ready to define M-hierarchy (m-hierarchy) as a family of $M_i = \{f : M'(f) \le j\}$.

Let us construct the analogous definition of L-hierarchy by replacing in the above definition M_x by L_x and changing line (5) to the following form (5'):

$$L_{x}\left(\underset{y\to\infty}{\text{liminf }}g\left(\overline{x},y\right)\right) = L_{x}\left(\underset{y\to\infty}{\text{lim sup }}g\left(\overline{x},y\right)\right) =$$

$$= L_{x}\left(\underset{y\to\infty}{\text{lim }}g\left(\overline{x},y\right)\right) = \max\left(L_{x}\left(f\right),L_{y}\left(f\right)\right) + 1.$$

For an **R**-recursive function f, let $L(f) = \max_i L_{x_i}(s)$ minimized over all expressions **s** that define **f** without using the **m**-operation.

Definition 3.2 The **L**-hierarchy is a family of $L_j = \{f : L(f) \le j\}$.

Let us add that in Definition 3.2 we use explicitly the operator $f(\overline{x}) = \lim_{y \to \infty} g(\overline{x}, y)$ to avoid its construction by other operators (\limsup lim inf), which would effect in a superficially higher class of a complexity of a function f.

As an obvious corollary from definitions we have the following statement.

Lemma 3.3 The classes M_0 and M_1 are identical.

A function $f \in L_0 = M_0$ will be called (by an analogy to the case of natural recursive functions) a primitive **R**-recursive function. After Moore [4] we can conclude that such functions as: -x, x+y, xy, x/y, e^x , $\ln x$, y^x , $\sin x$, $\cos x$ are primitive **R**-recursive.

We can give a few results on some levels of the limit hierarchy.

Lemma 3.4. The Kronecker d function, the signum function and absolute value belong to the first level (L_1) of limit hierarchy.

Proof. It is sufficient to take the following definitions [5]: hence d(0) = 1 and for all $x \neq 0$ we have d(x) = 0 let us define $d(x) = \liminf_{y \to \infty} \left(\frac{1}{1+x^2}\right)^y$. Now from the expression $\liminf_{y \to \infty} \arctan xy = \begin{cases} p/2, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ -p/2, & \text{if } x < 0, \end{cases}$ sgn $(x) = \frac{\liminf_{y \to \infty} \arctan xy}{2 \arctan 1}$ and $|x| = \operatorname{sgn}(x)x$.

We should be careful with definions of functions by cases:

$$\textbf{Lemma 3.5 For } h(\overline{x}) = \begin{cases} g_1(\overline{x}), & \text{if } f(\overline{x}) = 0, \\ g_2(\overline{x}), & \text{if } f(\overline{x}) = 1, \\ \mathbf{M} & \mathbf{M} \\ g_k(\overline{x}), & \text{if } f(\overline{x}) \ge k - 1 \end{cases} \text{ and } g_i \in L_{n_i} \text{ for all } 1 \le i \le k,$$

 $f \in L_m$ the function h belongs to $L_{\max(n_1,...n_k,m+1)}$

Proof. Let us see that
$$eq(x,y) = d(x-y) \in L_1$$
 and $ge(x,y) = \frac{\left(\operatorname{sgn}(x-y) + eq(x,y)\right)}{2} + \frac{1}{2} \in L_1$. Then of course $h(\overline{x}) = \sum_{i=1}^{k-1} g_i(\overline{x}) eq(f(\overline{x}),i-1) + g_k(\overline{x}) ge(f(\overline{x}),k-1)$.

Of course this result can be easily extended to other forms of definitions by cases.

Lemma 3.6 The function $\Theta(x)$ (equal to 1 if $x \ge 0$, otherwise 0), maximum $\max(x, y)$, square-wave function \mathbf{s} are in L_2 , the function p(x) such that p(x)=1 for $x \in [2n,2n+1]$ and p(x)=0 for $x \in [2n+1,2n+2]$ is in L_2 and the floor function |x| is in L_3 .

Proof. We give the proper definitions (from [6]) for these functions. Let

$$\Theta(x) = d(x - |x|),$$

$$\max(x, y) = xd(x - y) + (1 - d(x - y))[x\Theta(x - y) + y\Theta(y - x)],$$

$$s(x) = \Theta(\sin(px)).$$

The function
$$p(x)$$
 can be given as $s(x)\left(1-d\left(\sin\frac{(x-1)p}{2}\right)\right)$, so $p \in L_2$.

The floor function we can define by the auxiliary function w(0) = 0, $\partial_x w(x) = 2\Theta(-\sin(2px))$ as

$$\lfloor x \rfloor = \begin{cases} 2w(x/2) & \text{if } p(x) = 1, \\ 2w((x-1)/2) & \text{if } p(x) = 0. \end{cases}$$

From the above equation we have |x| in L_3 .

Let us recall that if $f: \mathbb{R}^n \to \mathbb{R}$ is an **R**-recursive function then the function $f_{iter}(i, \overline{x})$ is **R**-recursive, too.

Lemma 3.7 Let $f: \mathbb{R}^n \to \mathbb{R}$ belongs to the class L_i , then we have $f_{iter}: \mathbb{R}^{n+1} \to \mathbb{R}$ is in $L_{\max(2,j)}$.

Proof. The definitions, which were given by Moore [3] $f_{iter}(i, \overline{x}) = h(2i)$, where $h(0) = g(0) = \overline{x}$,

$$\partial_{t}g(t) = \left[f(h(t)) - h(t)\right]s(t),$$

$$\partial_{t}h(t) = \left[\frac{g(t) - h(t)}{r(t)}\right](1 - s(t)),$$

with s - a square wave function in L_2 and r(0) = 0, $\partial_t r(t) = 2s(t) - 1$, $r, s \in L_2$ give us the desirable statement. \square

Lemma 3.8 The \mathbf{R}^l -recursive functions $g_2: R^2 \to R$, $g_2^1, g_2^2: R \to R$ such that $(\forall x, y \in R) g_2^1 (g_2(x, y)) = x$, $(\forall x, y \in R) g_2^2 (g_2(x, y)) = y$, have the following properties: g_2 , g_2^1 are in L_{10} , g_2^2 is in L_{14} .

Proof. We have the auxiliary functions Γ_2 , Γ_2^1 , Γ_2^2 , which are coding and decoding functions in the interval (0,1): $\Gamma_2(x,y) = c(x) + c(y)/10$, where

$$c(x) = \lim z(a(i,x))/10^{2i} + b(i,x)/10^{i}$$
,

and later $z(x) = \lim_{i \to \infty} z_{iter}(i, x)$,

$$\begin{split} z_{iter}^{\cdot} \left(i, a_{1} \dots a_{n} . a_{n+1} \dots \right) &= a_{1} \dots a_{n} 0 \dots a_{n+1} 0 . a_{n+i+1} \dots, \\ a \left(i, 0 . a_{1} a_{2} \dots a_{i} \dots \right) &= 0 . a_{1} \dots a_{i} \\ b \left(i, 0 . a_{1} a_{2} \dots a_{i} \dots \right) &= 0 . \mathbf{Q}_{1} \dots 0 \ a_{i+1} \dots, \end{split}$$

$$(z'(x)) = \begin{cases} 100 \lfloor x \rfloor + 10(x - \lfloor x \rfloor), & \text{if } \lfloor x \rfloor \neq x, \\ x, & \text{if } \lfloor x \rfloor = x; \end{cases} \in L_4, a, b \in L_4. \text{ Also } z_{iter} \text{ belongs}$$

to L_4 , hence $\Gamma_2(x,y) \in L_{10}$, decoding of the first element is described in the symmetric way so $\Gamma_2^1(x)$ is in L_{10} , but $\Gamma_2^2(x) = \Gamma_2^1(10 - \lfloor 10x \rfloor)$ so $\Gamma_2^2 \in L_{14}$.

The functions Γ_2 , Γ_2^1 , Γ_2^2 can be extended to all reals by one-to-one $f:(0,1) \to R \in L_0$ without the loss of their class. \square

The same method of coding and decoding by interlacing of ciphers (only the power of 10 should be changed) gives us the functions $g_n: R^n \to R$ and $g_n^i: R \to R$ for i = 1,...,n such that

$$(\forall i)(\forall x_1...,x_n \in R)g_n^i(g_n(x_1,...,x_n)) = x_i$$

in the same class: $g_n, g_n^1 \in L_{10}$ and $(\forall i > 1)g_n^i \in L_{14}$.

We finish this part with the important form of defining: a new function is given as a product of values f in some integer points.

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Lemma 3.9 There exists such constant $p \in N$ that for the function

$$\prod_{z=0}^{y} f(\overline{x}, z) = \begin{cases} f(\overline{x}, 0) f(\overline{x}, 1) ... f(\overline{x}, \lfloor y - 1 \rfloor), & \text{if } y \ge 1, \\ 1, & \text{if } 0 \le y < 1, \\ 0, & \text{if } y < 0, \end{cases}$$

if the function f is in the class L_m then $\prod_{z=0}^y f(\overline{x}, z)$ is in the class L_{m+p} (p is independent of m).

Proof. By the definitions

$$t(w) = g_{n+2}(g_{n+2}^{1,n}(w), g_{n+2}^{n+1}(w) + 1, f(g_{n+2}^{1,n}(w), g_{n+2}^{n+1}(w)) \cdot g_{n+2}^{n+2}(w))$$

and

$$S(\overline{x},z) = t_{\text{tier}}(s(\overline{x},0))...) = t_{\text{tier}}(\lfloor z \rfloor, g_{n+2}(\overline{x},0,1))$$

we get the property

$$\prod_{y=0}^{z} f(\overline{x}, y) = g_{n+2}^{n+2} (S(\overline{x}, z)).$$

From the defintion of the limit hierarchy we get $\prod_{y=0}^{z} f(\overline{x}, y) \in L_{m+38}$.

In the rest of the paper we will use the constant p as the number of limits used in the recursive defintion of the product $\prod_{y=0}^{z} f(\overline{x}, y)$ instead of the value 38. The above constructions are tedious and can be improved with a better approximation of p.

4. Main results

Now we are ready to formulate two theorems which demonstrate connections between L-hierarchy and M-hierarchy.

Theorem 4.1 Let $f: \mathbb{R}^n \to \mathbb{R}$ be an **R**-recursive function. Then if $f \in L_i$ then $f \in M_{10i}$.

Proof. We use a simple induction here. The case i=0 is given in Lemma 3.3. Now let us suppose that the thesis is true for i=n. Let $f \in L_{n+1}$ be defined as $f(\overline{x}) = \lim_{y\to\infty} g(\overline{x}, y)$ for $g \in L_n$. Then we can recall Theorem 4.2 from [6] which gives us the following result: to define f from g it is necessary to use at

most 10 *m*-operation. Hence for $g \in M_{10n}$ the function **f** satisfies $f \in M_{10n+10}$. Similar inferences hold for \liminf , \limsup .

Now we can give the result about the 'limit complexity' of the infimum operator m.

Lemma 4.2 If $f(\overline{x}, y): R^{n+1} \to R$ is in the class L_m then the function $g: \mathbb{R}^n \to \mathbb{R}$, $g(\overline{x}) = \mathbf{m}_y f(\overline{x}, y)$ is in the class L_{m+3p+9} is from Lemma 3.9.

Proof. Here we must employ the results from [6]. There we defined the function $g: R^n \to R$, $g(\overline{x}) = m_y f(\overline{x}, y)$ for $f(\overline{x}, y): R^{n+1} \to R$ (f - **R**-recursive) replacing the *m*-operator by limit operation. First we introduced the function

This the following receives the first we introduced the function
$$Z^f(\overline{x}, z) = \begin{cases} \inf_y \left\{ f : K^f(\overline{x}, y) = 0 \right\}, & \text{if } z = 0 \text{ and } \exists y K^f(\overline{x}, y) = 0, \\ \text{undefined} & \text{if } z = 0 \text{ and } \forall y K^f(\overline{x}, y) \neq 0, \\ 1 & \text{if } z \neq 0, \end{cases}$$

given in the following way:

defined by an integration

from
$$S_i^f(\overline{x},t) = \int y^2 \left(1 - h^f(\overline{x},(-1)^{i+1}y - 1/2,(-1)^{i+1}y + 1/2)\right) dy$$
, $i = 1,2$
from $h^f(\overline{x},a,b) = \liminf_{t \to \infty} \prod_{w=0}^{z+1} K^f(\overline{x},a+w\frac{b-a}{z})$ where K^f is the characteristic function of f .

Hence we can conclude that if K^f is in the L_s then Z_f is in the class L_{s+p+3} . Let us finish with the definition of the characteristic function of the infimum of zeros of f (see Theorem 4.2 from [5]

$$K_{\mathbf{m}}^{f}(y) = 1 - \lim_{a \to \infty} \lim_{b \to \infty} \lim_{z \to \infty} G^{f}(\overline{x}, z, a, b, y),$$

where $G^{f}(\bar{x},z,a,b,y)$ divides the interval [a,b] into $2^{\lfloor z \rfloor}$ equal subintervals and gives the value 1 for y from the subintervals, which contains the least zero of f in [a,b] and value 0 otherwise. Precisely for y from $\left| a,a + \frac{b-a}{2^{\lfloor z \rfloor}} \right|$

$$G^{f}(\overline{x}, z, a, b, y) = \begin{cases} 1, & \text{if } h^{f}(\overline{x}, a, a + \frac{b - a}{2^{\lfloor z \rfloor}}) = 0, \\ 0, & \text{otherwise} \end{cases}$$

$$G^{f}\left(\overline{x},z,a,b,y\right) = \begin{cases} 1, & \text{if } h^{f}\left(\overline{x},a,a + \frac{b-a}{2^{\lfloor z \rfloor}}\right) = 0, \\ 0, & \text{otherwise} \end{cases}$$
 for $y \in \left(a + \frac{(k-1)(b-a)}{2^{\lfloor z \rfloor}}, a + \frac{k(b-a)}{2^{\lfloor z \rfloor}}\right)$ (where $k = 2,3,...,2^{n}$) we have:

$$G^{f}(\overline{x}, z, a, b, y) = \begin{cases} 1, & \text{if } \prod_{i=1}^{k-1} h^{f}\left(\overline{x}, a + \frac{(i-1)(b-a)}{2^{\lfloor z \rfloor}}, a + \frac{i(b-a)}{2^{\lfloor z \rfloor}}\right) \neq 0 \\ & \wedge h^{f}\left(\overline{x}, a + \frac{(k-1)(b-a)}{2^{\lfloor z \rfloor}}, a + \frac{k(b-a)}{2^{\lfloor z \rfloor}}\right) = 0, \\ 0, & \text{otherwise} \end{cases}$$

and for $Y \notin [A, B]$ the function g_x^f is equal to 2.

The definition of G_f is given by the cases with respect to the value of the expression given by $\prod h^f$, since for $f \in L_m$, the function $h_f \in L_{m+p+2}$ and $G^f \in L_{m+2p+3}$. Then we have $K_m^f \in L_{m+2p+6}$. Now we must use the function K_m^f in the same way as K^f which gives us Z_f in the class L_{m+3p+9} . The final definition of $g(\overline{x}) = \mathbf{m}_y f(\overline{x}, y)$ ([5] Theorem 4.3) given below

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$$g(\overline{x}) = \begin{cases} Z^{f^+}(\overline{x},0) - Z^{f^-}(\overline{x},0), & \text{if } S^{f^+}(\overline{x}) < \frac{1}{12} \land S^{f^-}(\overline{x}) < \frac{1}{12}, \\ Z^{f^+}(\overline{x},0), & \text{if } \left(S^{f^+}(\overline{x}) \ge \frac{1}{12} \land S^{f^-}(\overline{x}) < \frac{1}{12}\right) \end{cases}$$
or
$$\left(S^{f^+}(\overline{x}) < \frac{1}{12} \land S^{f^-}(\overline{x}) < \frac{1}{12}\right)$$

$$\wedge Z^{f^+}(\overline{x},0) < Z^{f^-}(\overline{x},0), \qquad \text{if } \left(S^{f^+}(\overline{x}) < \frac{1}{12} \land S^{f^-}(\overline{x}) \ge \frac{1}{12}\right) \end{cases}$$
or
$$\left(S^{f^+}(\overline{x}) < \frac{1}{12} \land S^{f^-}(\overline{x}) < \frac{1}{12}\right)$$
or
$$\left(S^{f^+}(\overline{x}) < \frac{1}{12} \land S^{f^-}(\overline{x}) < \frac{1}{12}\right)$$

$$\wedge Z^{f^+}(\overline{x},0) \ge Z^{f^-}(\overline{x},0), \qquad y \ge 0, \\ 1, \qquad y < 0; \qquad f^-(\overline{x},y) = \begin{cases} f(\overline{x},-y), & y > 0, \\ 1, & y \le 0; \end{cases} \end{cases}$$
 remains the class of g identical to the class of Z^f , i.e. $g \in L$

class of g identical to the class of Z^f , i.e. $g \in L_{m+3p+9}$.

Theorem 4.3 Let $f: \mathbb{R}^n \to \mathbb{R}$ be an **R**-recursive function. Then for all $i \ge 0$ if $f \in M_i$ then $f \in L_{(3p+9)i}$.

The above statement is a simple consequence of the fact $M_0 = L_0$ and Lemma 4.2.

5. Conclusions

In the paper we give the first rough approximation of 'a complexity' of limit operations in the terms of the *m*-operator and conversely. The results, interpreted in the intuitional way, can suggest what kind of connection exists between infinite limits and a *m*-operator.

We also establish the proper relation between the levels of the limit hierarchy and m-hierarchy. Let us point out that in consequence we may investigate analogies which exist for the limit hierarchy (also *m*-hierarchy) and Baire classes Pobrane z czasopisma Annales AI- Informatica http://ai.annales.umcs.pl Data: 06/12/2025 21:11:15

[7]. Also the kind of a connection between the $\sum_{n=0}^{\infty}$ measurable functions and \mathbf{R} -recursive functions is an open problem.

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